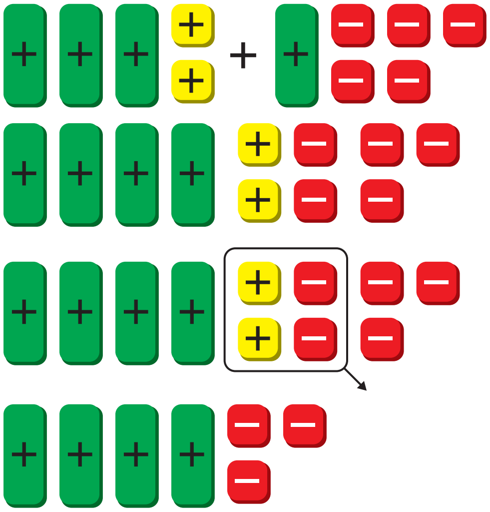
# 7.1 Adding and Subtracting Polynomials

## Essential Question

How can you add and subtract polynomials?

### EXPLORATION 1 Adding Polynomials

**Work with a partner.** Write the expression modeled by the algebra tiles in each step.



**Step 1** $(3x + 2) + (x - 5)$

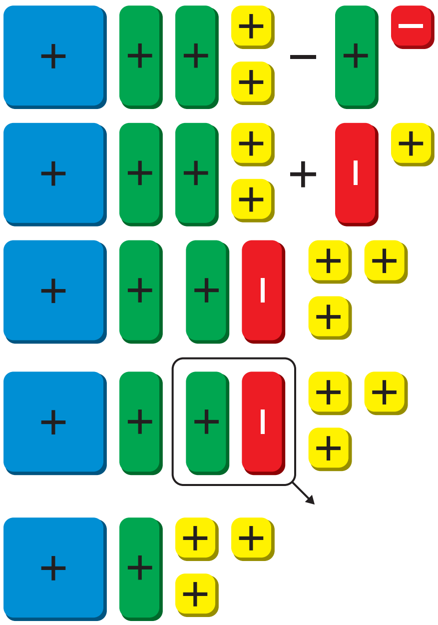
**Step 2**

**Step 3**

**Step 4**

### EXPLORATION 2 Subtracting Polynomials

**Work with a partner.** Write the expression modeled by the algebra tiles in each step.



**Step 1** $(x^2 + 2x + 2) - (x - 1)$

**Step 2**

**Step 3**

**Step 4**

**Step 5**

**REASONING ABSTRACTLY**

To be proficient in math, you need to represent a given situation using symbols.

## Communicate Your Answer

**3.** How can you add and subtract polynomials?

**4.** Use your methods in Question 3 to find each sum or difference.

**a.** $(x^2 + 2x - 1) + (2x^2 - 2x + 1)$ **b.** $(4x + 3) + (x - 2)$

**c.** $(x^2 + 2) - (3x^2 + 2x + 5)$ **d.** $(2x - 3x) - (x^2 - 2x + 4)$

# 7.1 Lesson

## Core Vocabulary

monomial, *p. 358*
degree of a monomial, *p. 358*
polynomial, *p. 359*
binomial, *p. 359*
trinomial, *p. 359*
degree of a polynomial, *p. 359*
standard form, *p. 359*
leading coefficient, *p. 359*
closed, *p. 360*

## What You Will Learn

- Find the degrees of monomials.
- Classify polynomials.
- Add and subtract polynomials.
- Solve real-life problems.

## Finding the Degrees of Monomials

A **monomial** is a number, a variable, or the product of a number and one or more variables with whole number exponents.

The **degree of a monomial** is the sum of the exponents of the variables in the monomial. The degree of a nonzero constant term is 0. The constant 0 does not have a degree.

| Monomial | Degree |
|---|---|
| 10 | 0 |
| $3x$ | 1 |
| $\frac{1}{2}ab^2$ | $1 + 2 = 3$ |
| $-1.8m^5$ | 5 |

| Not a monomial | Reason |
|---|---|
| $5 + x$ | A sum is not a monomial. |
| $\frac{2}{n}$ | A monomial cannot have a variable in the denominator. |
| $4^a$ | A monomial cannot have a variable exponent. |
| $x^{-1}$ | The variable must have a whole number exponent. |

### EXAMPLE 1 Finding the Degrees of Monomials

Find the degree of each monomial.

**a.** $5x^2$ **b.** $-\frac{1}{2}xy^3$ **c.** $8x^3y^3$ **d.** $-3$

**SOLUTION**

**a.** The exponent of $x$ is 2.

► So, the degree of the monomial is 2.

**b.** The exponent of $x$ is 1, and the exponent of $y$ is 3.

► So, the degree of the monomial is $1 + 3$, or 4.

**c.** The exponent of $x$ is 3, and the exponent of $y$ is 3.

► So, the degree of the monomial is $3 + 3$, or 6.

**d.** You can rewrite $-3$ as $-3x^0$.

► So, the degree of the monomial is 0.

## Monitoring Progress

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**Find the degree of the monomial.**

**1.** $-3x^4$ **2.** $7c^3d^2$ **3.** $\frac{5}{3}y$ **4.** $-20.5$

## Classifying Polynomials

### Core Concept

**Polynomials**

A **polynomial** is a monomial or a sum of monomials. Each monomial is called a *term* of the polynomial. A polynomial with two terms is a **binomial**. A polynomial with three terms is a **trinomial**.

| Binomial | Trinomial |
|---|---|
| $5x + 2$ | $x^2 + 5x + 2$ |

The **degree of a polynomial** is the greatest degree of its terms. A polynomial in one variable is in **standard form** when the exponents of the terms decrease from left to right. When you write a polynomial in standard form, the coefficient of the first term is the **leading coefficient**.

leading coefficient → degree → constant term

$$2x^3 + x^2 - 5x + 12$$

### EXAMPLE 2 Writing a Polynomial in Standard Form

Write $15x - x^3 + 3$ in standard form. Identify the degree and leading coefficient of the polynomial.

**SOLUTION**

Consider the degree of each term of the polynomial.

Degree is 1. → Degree is 3. → Degree is 0.

$$15x - x^3 + 3$$

▶ You can write the polynomial in standard form as $-x^3 + 15x + 3$. The greatest degree is 3, so the degree of the polynomial is 3, and the leading coefficient is $-1$.

### EXAMPLE 3 Classifying Polynomials

Write each polynomial in standard form. Identify the degree and classify each polynomial by the number of terms.

**a.** $-3z^4$ **b.** $4 + 5x^2 - x$ **c.** $8q + q^5$

**SOLUTION**

| Polynomial | Standard Form | Degree | Type of Polynomial |
|---|---|---|---|
| **a.** $-3z^4$ | $-3z^4$ | 4 | monomial |
| **b.** $4 + 5x^2 - x$ | $5x^2 - x + 4$ | 2 | trinomial |
| **c.** $8q + q^5$ | $q^5 + 8q$ | 5 | binomial |

### Monitoring Progress

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**Write the polynomial in standard form. Identify the degree and leading coefficient of the polynomial. Then classify the polynomial by the number of terms.**

**5.** $4 - 9z$ **6.** $t^2 - t^3 - 10t$ **7.** $2.8x + x^3$

## Adding and Subtracting Polynomials

A set of numbers is **closed** under an operation when the operation performed on any two numbers in the set results in a number that is also in the set. For example, the set of integers is closed under addition, subtraction, and multiplication. This means that if $a$ and $b$ are two integers, then $a + b$, $a - b$, and $ab$ are also integers.

The set of polynomials is closed under addition and subtraction. So, the sum or difference of any two polynomials is also a polynomial.

To add polynomials, add like terms. You can use a vertical or a horizontal format.

### EXAMPLE 4 Adding Polynomials

Find the sum.

**a.** $(2x^3 - 5x^2 + x) + (2x^2 + x^3 - 1)$ **b.** $(3x^2 + x - 6) + (x^2 + 4x + 10)$

**SOLUTION**

**a. Vertical format:** Align like terms vertically and add.

$$\begin{array}{r} 2x^3 - 5x^2 + x \phantom{{}-1} \\ +\ \ x^3 + 2x^2 \phantom{{}+x} - 1 \\ \hline 3x^3 - 3x^2 + x - 1 \end{array}$$

▶ The sum is $3x^3 - 3x^2 + x - 1$.

**b. Horizontal format:** Group like terms and simplify.

$$(3x^2 + x - 6) + (x^2 + 4x + 10) = (3x^2 + x^2) + (x + 4x) + (-6 + 10)$$
$$= 4x^2 + 5x + 4$$

▶ The sum is $4x^2 + 5x + 4$.

**STUDY TIP**

When a power of the variable appears in one polynomial but not the other, leave a space in that column, or write the term with a coefficient of 0.

To subtract a polynomial, add its opposite. To find the opposite of a polynomial, multiply each of its terms by $-1$.

### EXAMPLE 5 Subtracting Polynomials

Find the difference.

**a.** $(4n^2 + 5) - (-2n^2 + 2n - 4)$ **b.** $(4x^2 - 3x + 5) - (3x^2 - x - 8)$

**SOLUTION**

**a. Vertical format:** Align like terms vertically and subtract.

$$\begin{array}{r} 4n^2 \phantom{{}+2n} + 5 \\ -\ (-2n^2 + 2n - 4) \\ \hline \end{array} \quad \Rightarrow \quad \begin{array}{r} 4n^2 \phantom{{}-2n} + 5 \\ +\ \ 2n^2 - 2n + 4 \\ \hline 6n^2 - 2n + 9 \end{array}$$

▶ The difference is $6n^2 - 2n + 9$.

**b. Horizontal format:** Group like terms and simplify.

$$(4x^2 - 3x + 5) - (3x^2 - x - 8) = 4x^2 - 3x + 5 - 3x^2 + x + 8$$
$$= (4x^2 - 3x^2) + (-3x + x) + (5 + 8)$$
$$= x^2 - 2x + 13$$

▶ The difference is $x^2 - 2x + 13$.

**COMMON ERROR**

Remember to multiply *each* term of the polynomial by $-1$ when you write the subtraction as addition.

## Monitoring Progress

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**Find the sum or difference.**

**8.** $(b - 10) + (4b - 3)$

**9.** $(x^2 - x - 2) + (7x^2 - x)$

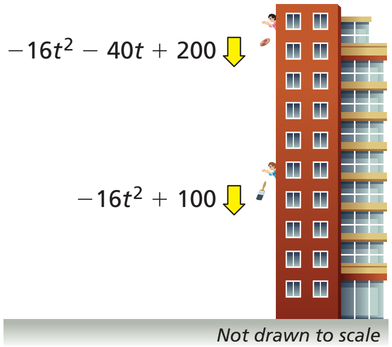
**10.** $(p^2 + p + 3) - (-4p^2 - p + 3)$

**11.** $(-k + 5) - (3k^2 - 6)$

## Solving Real-Life Problems

### EXAMPLE 6 Solving a Real-Life Problem

A penny is thrown straight down from a height of 200 feet. At the same time, a paintbrush is dropped from a height of 100 feet. The polynomials represent the heights (in feet) of the objects after $t$ seconds.




**a.** Write a polynomial that represents the distance between the penny and the paintbrush after $t$ seconds.

**b.** Interpret the coefficients of the polynomial in part (a).

### SOLUTION

**a.** To find the distance between the objects after $t$ seconds, subtract the polynomials.

| | | | |
|---|---|---|---|
| **Penny** | $-16t^2 - 40t + 200$ | | $-16t^2 - 40t + 200$ |
| **Paintbrush** | $-\ (-16t^2 \qquad + 100)$ | ➡ | $+\quad 16t^2 \qquad - 100$ |
| | | | $-40t + 100$ |

▶ The polynomial $-40t + 100$ represents the distance between the objects after $t$ seconds.

**b.** When $t = 0$, the distance between the objects is $-40(0) + 100 = 100$ feet. So, the constant term 100 represents the distance between the penny and the paintbrush when both objects begin to fall.

As the value of $t$ increases by 1, the value of $-40t + 100$ decreases by 40. This means that the objects become 40 feet closer to each other each second. So, $-40$ represents the amount that the distance between the objects changes each second.

## Monitoring Progress

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**12. WHAT IF?** The polynomial $-16t^2 - 25t + 200$ represents the height of the penny after $t$ seconds.

**a.** Write a polynomial that represents the distance between the penny and the paintbrush after $t$ seconds.

**b.** Interpret the coefficients of the polynomial in part (a).

# 7.1 Exercises

Dynamic Solutions available at *BigIdeasMath.com*

## Vocabulary and Core Concept Check

1. **VOCABULARY** When is a polynomial in one variable in standard form?

2. **OPEN-ENDED** Write a trinomial in one variable of degree 5 in standard form.

3. **VOCABULARY** How can you determine whether a set of numbers is closed under an operation?

4. **WHICH ONE DOESN'T BELONG?** Which expression does *not* belong with the other three? Explain your reasoning.

| | | | |
|---|---|---|---|
| $a^3 + 4a$ | $x^2 - 8^x$ | $b - 2^{-1}$ | $-\frac{\pi}{3} + 6y^8z$ |

## Monitoring Progress and Modeling with Mathematics

**In Exercises 5–12, find the degree of the monomial.** *(See Example 1.)*

5. $4g$
6. $23x^4$
7. $-1.75k^2$
8. $-\frac{4}{9}$
9. $s^8t$
10. $8m^2n^4$
11. $9xy^3z^7$
12. $-3q^4rs^6$

**In Exercises 13–20, write the polynomial in standard form. Identify the degree and leading coefficient of the polynomial. Then classify the polynomial by the number of terms.** *(See Examples 2 and 3.)*

13. $6c^2 + 2c^4 - c$
14. $4w^{11} - w^{12}$
15. $7 + 3p^2$
16. $8d - 2 - 4d^3$
17. $3t^8$
18. $5z + 2z^3 + 3z^4$
19. $\pi r^2 - \frac{5}{7}r^8 + 2r^5$
20. $\sqrt{7}n^4$

21. **MODELING WITH MATHEMATICS** The expression $\frac{4}{3}\pi r^3$ represents the volume of a sphere with radius *r*. Why is this expression a monomial? What is its degree?

22. **MODELING WITH MATHEMATICS** The amount of money you have after investing \$400 for 8 years and \$600 for 6 years at the same interest rate is represented by $400x^8 + 600x^6$, where *x* is the growth factor. Classify the polynomial by the number of terms. What is its degree?

**In Exercises 23–30, find the sum.** *(See Example 4.)*

23. $(5y + 4) + (-2y + 6)$
24. $(-8x - 12) + (9x + 4)$
25. $(2n^2 - 5n - 6) + (-n^2 - 3n + 11)$
26. $(-3p^3 + 5p^2 - 2p) + (-p^3 - 8p^2 - 15p)$
27. $(3g^2 - g) + (3g^2 - 8g + 4)$
28. $(9r^2 + 4r - 7) + (3r^2 - 3r)$
29. $(4a - a^3 - 3) + (2a^3 - 5a^2 + 8)$
30. $(s^3 - 2s - 9) + (2s^2 - 6s^3 + s)$

**In Exercises 31–38, find the difference.** *(See Example 5.)*

31. $(d - 9) - (3d - 1)$
32. $(6x + 9) - (7x + 1)$
33. $(y^2 - 4y + 9) - (3y^2 - 6y - 9)$
34. $(4m^2 - m + 2) - (-3m^2 + 10m + 4)$
35. $(k^3 - 7k + 2) - (k^2 - 12)$
36. $(-r - 10) - (-4r^3 + r^2 + 7r)$

37. $(t^4 - t^2 + t) - (12 - 9t^2 - 7t)$

38. $(4d - 6d^3 + 3d^2) - (10d^3 + 7d - 2)$

**ERROR ANALYSIS In Exercises 39 and 40, describe and correct the error in finding the sum or difference.**

39.

✗
$$\begin{aligned}(x^2 + x) - (2x^2 - 3x) &= x^2 + x - 2x^2 - 3x \\ &= (x^2 - 2x^2) + (x - 3x) \\ &= -x^2 - 2x\end{aligned}$$

40.

✗
$$\begin{array}{r} x^3 - 4x^2 + 3 \\ + \; -3x^3 + 8x \;\; - 2 \\ \hline -2x^3 + 4x^2 + 1 \end{array}$$

41. **MODELING WITH MATHEMATICS** The cost (in dollars) of making $b$ bracelets is represented by $4 + 5b$. The cost (in dollars) of making $b$ necklaces is represented by $8b + 6$. Write a polynomial that represents how much more it costs to make $b$ necklaces than $b$ bracelets.

42. **MODELING WITH MATHEMATICS** The number of individual memberships at a fitness center in $m$ months is represented by $142 + 12m$. The number of family memberships at the fitness center in $m$ months is represented by $52 + 6m$. Write a polynomial that represents the total number of memberships at the fitness center.

**In Exercises 43–46, find the sum or difference.**

43. $(2s^2 - 5st - t^2) - (s^2 + 7st - t^2)$

44. $(a^2 - 3ab + 2b^2) + (-4a^2 + 5ab - b^2)$

45. $(c^2 - 6d^2) + (c^2 - 2cd + 2d^2)$

46. $(-x^2 + 9xy) - (x^2 + 6xy - 8y^2)$

**REASONING In Exercises 47–50, complete the statement with *always*, *sometimes*, or *never*. Explain your reasoning.**

47. The terms of a polynomial are ________ monomials.

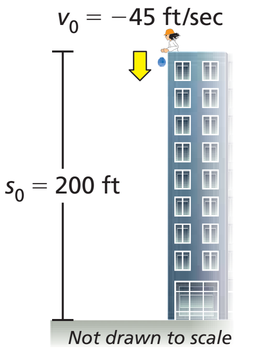
48. The difference of two trinomials is ________ a trinomial.

49. A binomial is ________ a polynomial of degree 2.

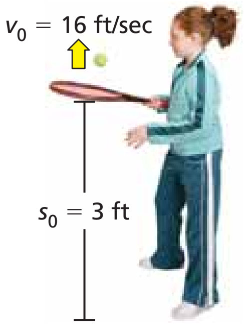
50. The sum of two polynomials is ________ a polynomial.

**MODELING WITH MATHEMATICS The polynomial $-16t^2 + v_0t + s_0$ represents the height (in feet) of an object, where $v_0$ is the initial vertical velocity (in feet per second), $s_0$ is the initial height of the object (in feet), and $t$ is the time (in seconds). In Exercises 51 and 52, write a polynomial that represents the height of the object. Then find the height of the object after 1 second.**

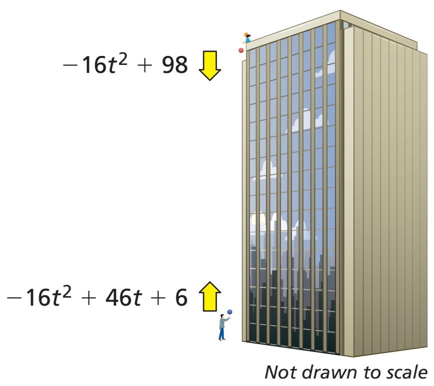
51. You throw a water balloon from a building.




52. You bounce a tennis ball on a racket.




53. **MODELING WITH MATHEMATICS** You drop a ball from a height of 98 feet. At the same time, your friend throws a ball upward. The polynomials represent the heights (in feet) of the balls after $t$ seconds. *(See Example 6.)*




a. Write a polynomial that represents the distance between your ball and your friend's ball after $t$ seconds.

b. Interpret the coefficients of the polynomial in part (a).

**37.** $(t^4 - t^2 + t) - (12 - 9t^2 - 7t)$

**38.** $(4d - 6d^3 + 3d^2) - (10d^3 + 7d - 2)$

**ERROR ANALYSIS** **In Exercises 39 and 40, describe and correct the error in finding the sum or difference.**

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$$\begin{aligned}(x^2 + x) - (2x^2 - 3x) &= x^2 + x - 2x^2 - 3x \\ &= (x^2 - 2x^2) + (x - 3x) \\ &= -x^2 - 2x\end{aligned}$$

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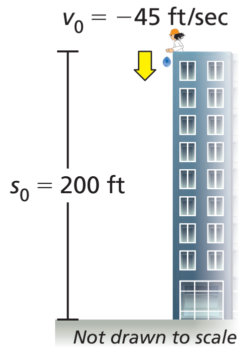


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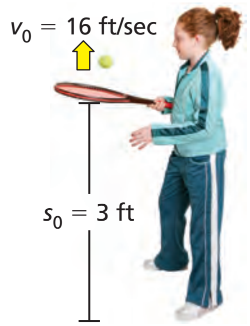


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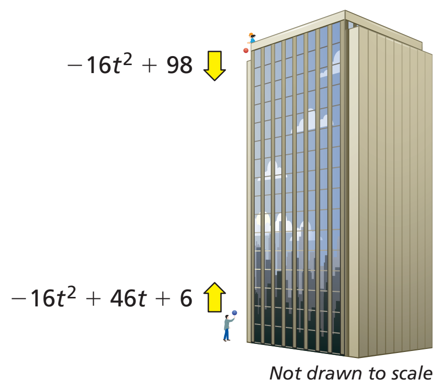


**a.** Before the balls reach the same height, write a polynomial that represents the distance between your ball and your friend's ball after $t$ seconds.

**b.** Interpret the coefficients of the polynomial in part (a).